Dynamic Optimal Transport using Divergence-Free decompositions

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In this talk we address the resolution of the dynamic optimal transport (OT) problem between 1D or 2D images in the fluid mechanics framework of Benamou-Brenier [1]. The numerical resolution of this dynamic formulation of OT, despite the successful application of proximal methods [2] is still computationally demanding. This is partly due to a space-time Laplace operator to be solved at each iteration, to project back to a divergence free space. We present several methods based on the Helmholtz-Hodge decomposition in order to enforce the divergence-free constraint throughout the iterations. We first prove that the functional we consider has better convexity properties on the set of constraints. Then we propose several formulations: in terms of minimal surface equation [3], using the primal-dual algorithm of Chambolle and Pock [4], or using a divergence-free wavelet decomposition [5].

References

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