Long time behavior of solutions of a discretized Fokker-Planck equation

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The exponential return to the equilibrium of the solutions of inhomogeneous kinetic equations in large time is now rather well-known thanks to the hypocoercive theory [2, 4, 3, 1]... The key is that one commutator with the velocity erases the degeneracy at $v = 0$ of the drift operator $v \cdot \nabla_x$.

Numerically, using schemes in long time requires special techniques, as discretizations often lead to error terms that depend exponentially on the final time, and are thus not relevant as they are.

We will discuss the choice of the discretizations of the differential operators appearing in the Fokker-Planck equation to ensure good numerical properties of the long time behavior, in the spirit of the already known results of hypocoercivity and exponential time decay in the continuous setting.

References

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