The ML-EM algorithm in continuum: sparse measure solutions

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The ML-EM algorithm aims at solving the maximum likelihood problem associated to a linear inverse problem $Ax = y$ with Poisson noise and nonnegative unknown x. It has been developed for medical imaging and is still used in Positron Emission Tomography [1]. The algorithm is known to produce "spiky" artefacts, as some pixels take arbitrarily large values along iterates. My presentation stems from a joint work with Olivier Verdier [2]. It will be focussed on the analysis of ML-EM *in continuum* and how it sheds light on this phenomenon. I will show that the right functional space is that of measures, and that in the unfavourable case when the linear problem $Ax = y$ does not have any solution, Dirac masses should be expected at the limit. I will also discuss the favourable case, and through simulations and a bit of convex analysis, I will show the link between theory and practice. Finally and if time permits, I'll present what variable controls the probability to be in the unfavourable case.

References

[1] L.A. Shepp, Y. Vardi, *Maximum likelihood reconstruction for emission tomography*, IEEE transactions on medical imaging 1.2 (1982), pp. 113122..

[2] C. P., O. Verdier, *The ML-EM algorithm in continuum: sparse measure solutions*, preprint arXiv:1909.01966, 2017.